to the sum of two squares, I have found the following complete sets of "eggs" for each nest.

In nest 1, where $n=1$ and $N=1$: 1, 2, 2. PROOF.—$1^2+2^2+2^2=(3^1)^2=9$.

In nest 2, where $n=2$ and $N=2$: 1, 4, 8; 7, 4, 4.

In nest 3, where $n=3$ and $N=5$: 7, 2, 26; 7, 14, 22; 23, 2, 14; 23, 10, 10; 25, 2, 10.

In nest 4, where $n=4$ and $N=14$:

1, 28, 76; 1, 44, 68; 17, 56, 56; 23, 16, 76; 23, 44, 64; 41, 16, 68; 41, 28, 64; 47, 16, 64; 49, 8, 64; 49, 32, 56; 55, 20, 56; 55, 40, 44; 65, 20, 44; 79, 8, 16.

In nest 5, where $n=5$ and $N=41$:

1, 22, 242; 17, 14, 242; 17, 46, 238; 17, 106, 218; 17, 134, 202; 31, 38, 238; 31, 158, 182; 47, 14, 238; 47, 154, 182; 49, 2, 238; 49, 158, 178; 95, 118, 190; 95, 50, 218; 95, 130, 182; 97, 46, 218; 97, 62, 214; 97, 94, 202; 97, 134, 178; 113, 22, 214; 113, 62, 206; 113, 74, 202; 113, 146, 158; 127, 22, 206; 127, 46, 202; 127, 106, 178; 127, 134, 158; 143, 50, 190; 143, 74, 182; 143, 122, 154; 145, 70, 182; 161, 2, 182; 161, 38, 178; 193, 22, 146; 193, 62, 134; 193, 70, 130; 209, 22, 122; 209, 38, 118; 223, 22, 94; 223, 62, 74; 239, 22, 38; 241, 22, 22.

In nest 6, where $n=6$ and $N=122$:

17, 136, 716; 17, 436, 584; 49, 236, 668; 49, 224, 692; 49, 128, 716; 49, 496, 532; 55, 196, 700; 73, 116, 716; 73, 44, 724; 79, 32, 724; 79, 172, 704; 79, 112, 716; 79, 308, 656; 79, 340, 640; 79, 460, 560; 89, 224, 688; 89, 416, 592; 103, 376, 616; 103, 424, 584; 119, 68, 716; 119, 196, 692; 119, 436, 572; 119, 484, 532; 137, 4, 716; 137, 236, 676; 143, 124, 704; 143, 284, 656; 175, 196, 680; 175, 104, 700; 185, 40, 704; 185, 296, 640; 199, 136, 688; 199, 248, 656; 199, 304, 632; 199, 376, 592; 239, 308, 616; 241, 4, 688; 241, 416, 548; 241, 128, 676; 241, 464, 508; 247, 116, 676; 247, 236, 644; 271, 32, 676; 271, 220, 640; 271, 208, 644; 271, 380, 560; 313, 56, 656; 313, 304, 584; 329, 92, 644; 329, 460, 460; 337, 56, 644; 337, 196, 616; 359, 56, 632; 359, 152, 616; 359, 248, 584; 359, 424, 472; 367, 236, 584; 367, 296, 556; 401, 172, 584; 401, 248, 556; 401, 296, 532; 401, 364, 488; 409, 376, 472; 409, 152, 584; 431, 68, 584; 431, 136, 572; 431, 296, 508; 431, 376, 452; 439, 172, 556; 439, 196, 548; 439, 236, 532; 439, 284, 508; 455, 104, 560; 455, 280, 496; 457, 116, 556; 457, 364, 436; 473, 304, 464; 497, 196, 496; 497, 224, 484; 521, 44, 508; 521, 100, 500; 521, 220, 460; 521, 236, 452; 521, 340, 380; 527, 196, 464; 527, 284, 516; 529, 40, 500; 529, 116, 488; 529, 200, 460; 529, 268, 424; 529, 288, 436; 529, 332, 376; 551, 112, 464; 551, 304, 368; 583, 136, 416; 583, 224, 376; 593, 4, 424; 593, 196, 376; 623, 44, 376; 623, 104, 364; 623, 236, 296; 625, 196, 320; 625, 220, 304; 631, 28, 364; 631, 196, 308; 649, 4, 332; 649, 124, 308; 649, 172, 284; 649, 196, 268; 655, 4, 320; 655, 100, 304; 655, 40, 296; 655, 104, 280; 689, 32, 236; 689, 116, 208; 695, 4, 220; 695, 100, 196; 719, 32, 116; 719, 44, 112; 721, 28, 104; 721, 40, 100; 721, 56, 92.

In order to obtain the multiple sets in the several nests, we multiply the prime sets of all the preceding nests by the necessary power of 3.

The multiple sets in nest 4 are 3^3 times the prime set of nest 1, 3^2 times the prime sets of nest 2, and 3 times those of nest 3, making 8 in all.

Also solved by *CHARLES C. CROSS.*

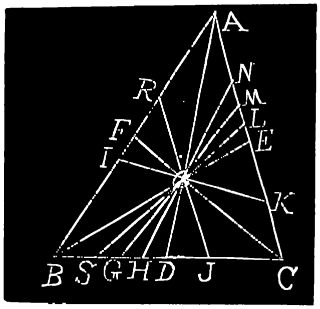
AVERAGE AND PROBABILITY.

56. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

Find the chance that the center of gravity of a triangle lies inside the triangle formed by three points taken at random within the triangle. [From *Williamson's Integral Calculus.*]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let ABC be the given triangle, D, E, F the mid-points of the sides, and O the center of gravity. Also let $DG=x$, DS, DH, DJ, FK, FL, FN, ER, or $EI=y$.



Now $FM=BM-\frac{1}{2}c=\dfrac{c(a-2x)}{2(a+6x)}$.

When the first point is on OG and the second on OH, OS, OJ, OK, OL, ON, OR, or OI, respectively, then the third point must fall on

$$MON=\frac{\triangle}{3}\left(\frac{4x}{a+6x}-\frac{4y}{a+6y}\right),\quad MOL=\frac{\triangle}{3}\left(\frac{4y}{a+6y}-\frac{4x}{a+6x}\right),$$

$$MARO=\frac{\triangle}{3}\left(\frac{4x}{a+6x}+\frac{4y}{a+6y}\right),\quad MAIO=\frac{\triangle}{3}\left(\frac{4x}{a+6x}+1-\frac{4y}{c+6y}\right),$$

$$MACSO=\frac{\triangle}{3}\left(\frac{4x}{a+6x}+1+\frac{4y}{c+6y}\right),\quad MBHO=\frac{\triangle}{3}\left(2-\frac{4x}{a+6x}-\frac{4y}{c+6y}\right),$$

$$MOJB=\frac{\triangle}{3}\left(\frac{4y}{b+6y}+1-\frac{4x}{a+6x}\right),\quad MOK=\frac{\triangle}{3}\left(1-\frac{4y}{b+6y}-\frac{4x}{a+6x}\right),$$

respectively.

The chance that the first is on OG is $dx/3a$; that the second is on OH, OS, or OJ is $dy/3a$; that it is on OK, OL, or ON is $dy/3c$; that it is on OR, or OI is $dy/3b$.

$$\therefore p=\frac{1}{27a^2}\left[\int_0^{\frac{1}{2}a}\left\{\int_0^x\left(\frac{4x}{a+6x}-\frac{4y}{a+6y}\right)dy+\int_x^{\frac{1}{2}a}\left(\frac{4y}{a+6y}-\frac{4x}{a+6x}\right)dy\right\}dx\right.$$

$$\left.+\int_0^{\frac{1}{2}a}\int_0^{\frac{1}{2}a}\left(\frac{4x}{a+6x}+\frac{4y}{a+6y}\right)dxdy\right]+\frac{1}{27ac}\left[\int_0^{\frac{1}{2}a}\int_0^{\frac{1}{2}c}\left(\frac{4x}{a+6x}+1-\frac{4y}{c+6y}\right)dxdy\right.$$

$$\left.+\int_0^{\frac{1}{2}a}\left\{\int_0^{FM}\left(\frac{4x}{a+6x}+1+\frac{4y}{c+6y}\right)dy+\int_{FM}^{\frac{1}{2}c}\left(2-\frac{4x}{a+6x}-\frac{4y}{c+6y}\right)dy\right\}dx\right]$$

$$+\frac{1}{27ab}\left[\int_0^{\frac{1}{2}a}\int_0^{\frac{1}{2}b}\left(\frac{4y}{b+6y}+1-\frac{4x}{a+6x}\right)dxdy+\int_0^{\frac{1}{2}a}\int_0^{\frac{1}{2}b}\left(1-\frac{4y}{b+6y}-\frac{4x}{a+6x}\right)dxdy\right]$$

$$\therefore p=\frac{1}{27a}\int_0^{\frac{1}{2}a}\left[\frac{5}{2}+\frac{a-34x}{6(a+6x)}+\frac{4x(a-2x)}{(a+6x)^2}+\frac{4}{9}\log\left(\frac{a+6x}{a}\right)-\frac{2}{9}\log 4\right]dx$$

$$=\frac{1}{27}\left(\frac{1}{3}+\frac{5}{9}\log 4\right).$$

Since the first point can fall in any of the six portions into which the medians divide the triangle, the required probability is $P=6p$.

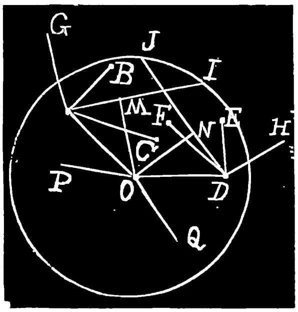
$\therefore\ P=\frac{6}{27}(\frac{1}{3}+\frac{5}{9}\log 4)=\frac{1}{27}(2+\frac{10}{3}\log 4=\frac{2}{27}(1+\frac{10}{3}\log 2)$.

91. **Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.**

Six points A, B, C, D, E, F, are taken at random on the surface of a sphere. Find the chance that the plane through A, B, C intersects the plane through D, E, F within the sphere.

Solution by the PROPOSER.

Let AI, DJ be the diameters of the sections of the sphere made by the planes through A, B, C and D, E, F; M and N their centers, O the center of the sphere, OP a line such that AB is parallel to the plane MOP, OQ a line such that DE is parallel to the plane NOQ. Draw AG, DH perpendicular to AI, DJ, respectively.



Let $AO=r$, $\angle AOM=\theta$, $\angle DON=\varphi$, $\angle GAC=\psi$, $\angle GAB=\delta$, $\angle HDF=\mu$, $\angle HDE=\nu$, $\angle MOP=\lambda$, the angle the plane MOP makes with some fixed plane through $OP=\rho$, $\angle NOQ=\gamma$, and the diedral angle $MOQN=\eta$.

An element of surface at A is $4\pi r^2\sin\theta d\theta$; at D, $4\pi r^2\sin\varphi d\varphi$; at C, $4r^2\sin\theta\sin\psi d\psi d\lambda$; at B, $4r^2\sin\theta\sin(\psi-\delta)\sin\lambda\sin\delta d\delta d\rho$; at F, $4r^2\sin\varphi\sin\mu d\mu d\gamma$; at E, $4r^2\sin\varphi\sin(\mu-\nu)\sin\gamma\sin\nu d\nu d\eta$.

The limits of θ are 0 and $\frac{1}{2}\pi$; of φ, 0 and $\frac{1}{2}\pi$; of ψ, 0 and π; of δ, 0 and ψ; of μ, 0 and π; of ν, 0 and μ; of λ, 0 and π; of γ. $\pm(\theta-\varphi)$ and $\theta+\varphi$ (the double sign being taken $+$ when $\theta>\varphi$, and $-$ when $\theta<\varphi$); of ρ, 0 and 2π; of η, 0 and 2π.

Since the whole number of ways six points can be taken in the surface of the sphere is $(4\pi r^2)^6$ the required chance is

$$p=\frac{1}{(4\pi r^2)^6}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\pi}\int_0^{\pi}\int_0^{\psi}\int_0^{\mu}\int_0^{\pi}\int_{\pm(\theta-\psi)}^{\theta+\psi}\int_0^{2\pi}\int_0^{2\pi}4\pi r^2\sin\theta.d\theta 4\pi r^2$$

$$\sin\varphi d\varphi.4r^2\sin\theta\sin\psi d\psi d\lambda.4r^2\sin\varphi\sin\mu d\mu d\gamma.4r^2\sin\theta\sin(\psi-\delta)\sin\lambda\sin\delta d\delta d\rho$$

$$\times 4r^2\sin\varphi\sin(\mu-\nu)\sin\gamma\sin\nu d\nu d\gamma$$

$$=\frac{2}{\pi^3}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\pi}\int_0^{\pi}\int_0^{\psi}\int_0^{\mu}\int_0^{\pi}\int_{\pm(\theta-\psi)}^{\theta+\psi}\int_0^{2\pi}\sin^3\theta\sin^3\varphi\sin\psi\sin\mu\sin(\psi-\delta)$$

$$\sin(\mu-\nu)\sin\delta\sin\nu\sin\lambda\sin\gamma d\theta d\varphi d\psi d\mu d\delta d\nu d\gamma d\rho$$

$$=\frac{4}{\pi^2}\int_0^{\frac{1}{2}\pi}\int_0^{\frac{1}{2}\pi}\int_0^{\pi}\int_0^{\pi}\int_0^{\psi}\int_0^{\mu}\int_0^{\pi}\int_{\pm(\theta-\psi)}^{\theta+\psi}\sin^3\theta\sin^3\varphi\sin\psi\sin\mu\sin(\psi-\delta)$$